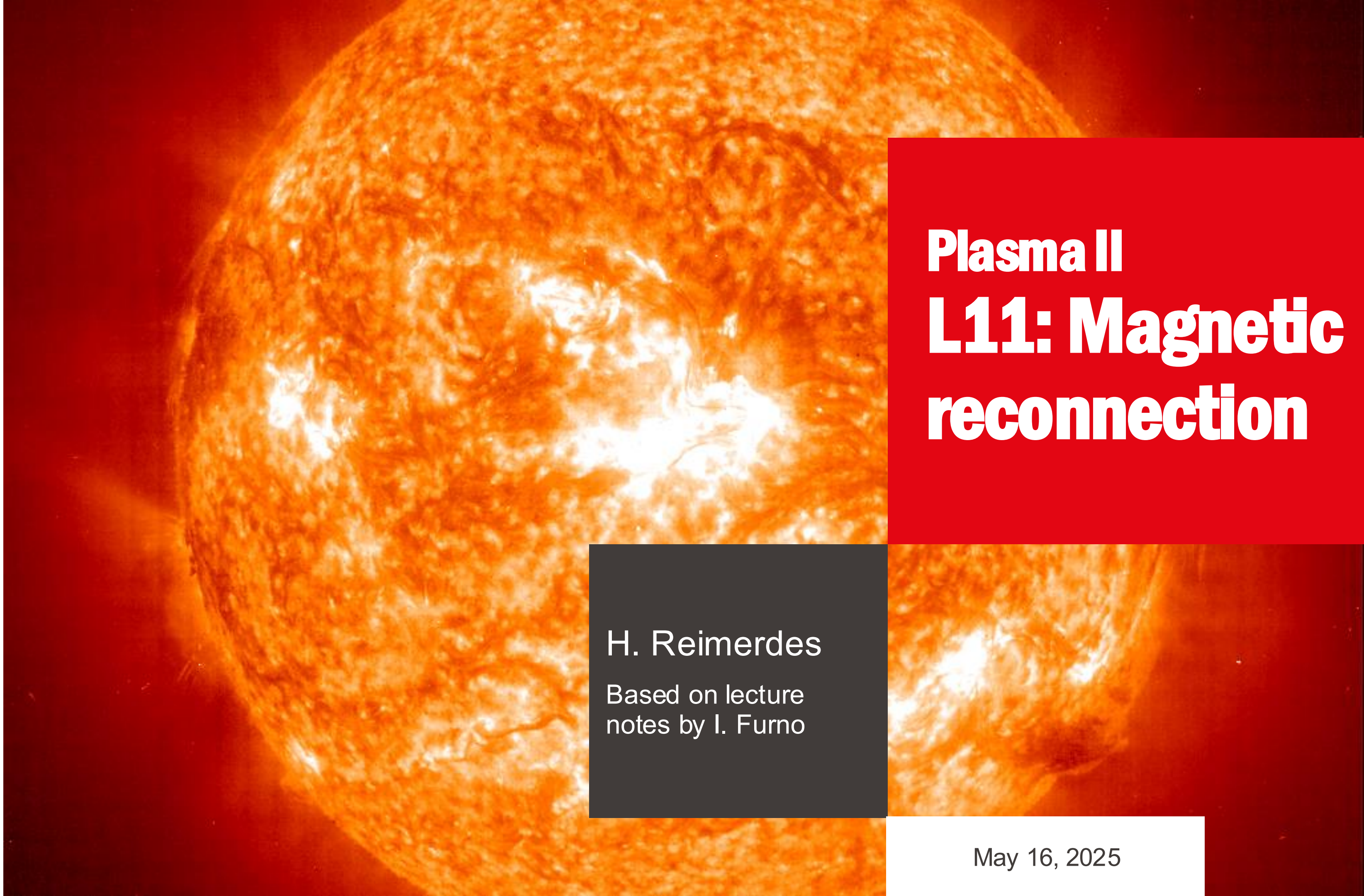


EPFL

Plasma II
L11: Magnetic reconnection

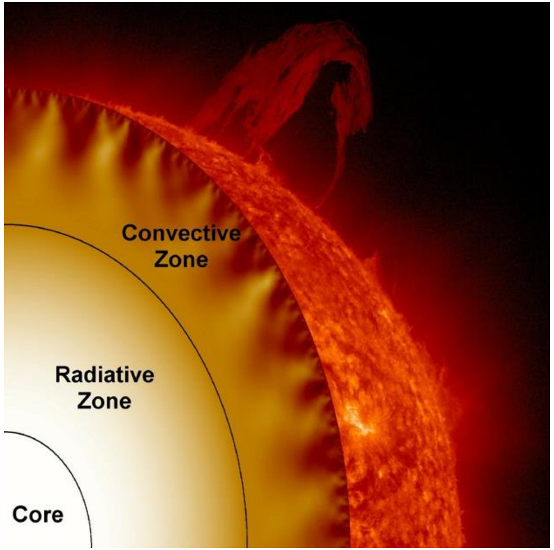
H. Reimerdes

Based on lecture notes by I. Furno

May 16, 2025

Swiss Plasma Center

Content of astrophysics module



- The sun's nuclear energy source
- Transport processes

} L9

- The structure of its magnetic field
- The solar dynamo

} L10

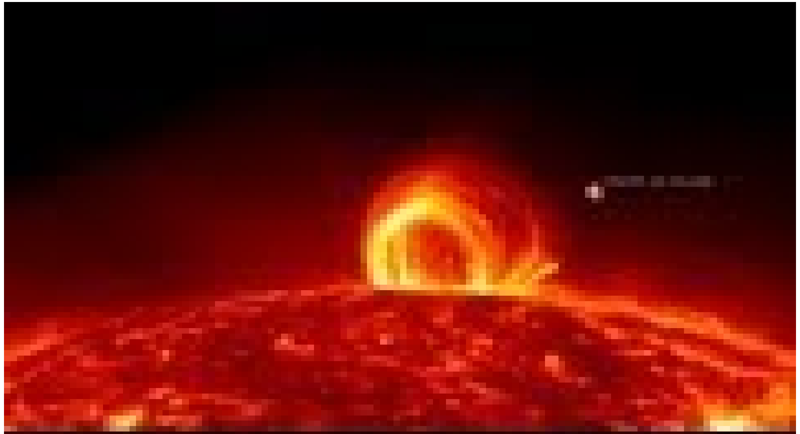
- Magnetic reconnection
- Solar wind

} L11

➢ See also EPFL MOOC "Plasma physics: Applications" #4c-d
https://learning.edx.org/course/course-v1:EPFLx+PlasmaApplicationX+1T_2018/home

- N. Meyer-Vernet, "Basics of the solar wind", Cambridge Atmospheric and Space Science Series, Section 3

Solar flares



http://www.youtube.com/watch?v=GrnGi-q6iWc

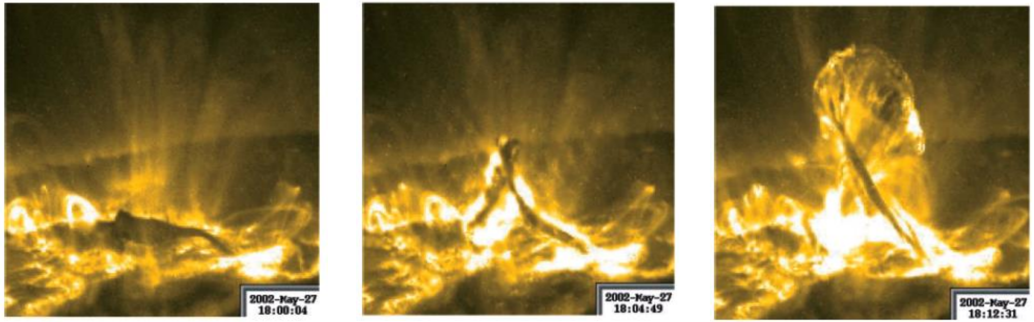
Classification by peak X-ray (100-800pm) in Watts/m^2

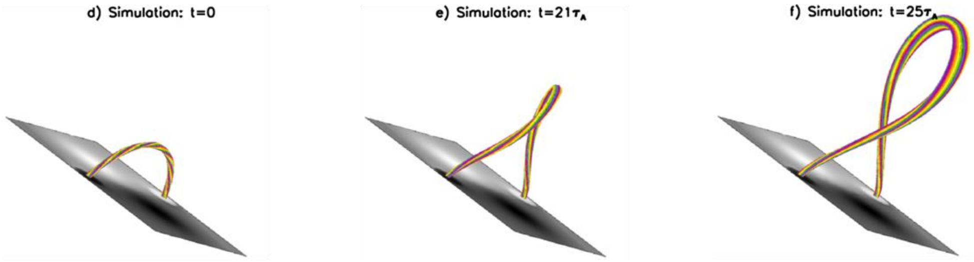
A	$< 10^{-7}$
B	10^{-7} - 10^{-6}
C	10^{-6} - 10^{-5}
M	10^{-5} - 10^{-4}
X	10^{-4} - 10^{-3}

- A sudden flash of brightness observed over the Sun's 'surface', which is interpreted as a large energy release (up to 6×10^{25} J)
- Often followed by a coronal mass ejection (CME)
- Main features
  - Accumulation of energy for long period and then sudden release
  - Particle acceleration → heating of the corona

Flares are flux ropes that can be subject to MHD instabilities

- On May 27 2002, TRACE observed an M2 that was not followed by a CME
  - Observations in the extreme ultraviolet (~20nm)
- The flare is accompanied by a filament eruption → **kink instability (see L10-E2)**

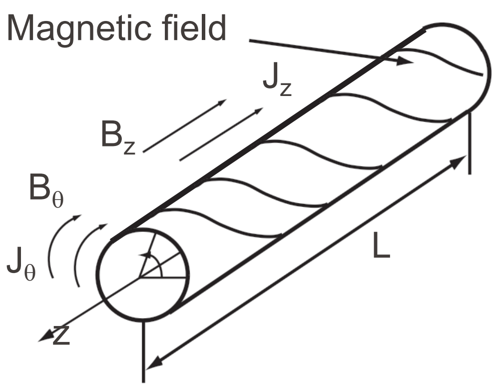


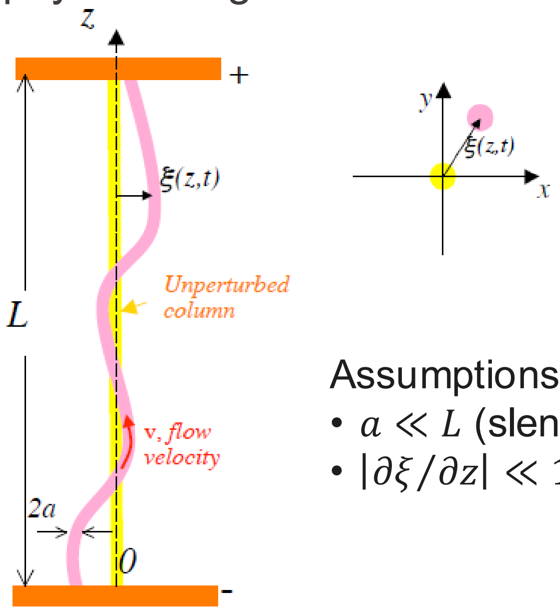


Török & Kliem, 2005

Kink instability

- Kruskal-Srafranov current threshold for screw pinches
- Consider a slender column → gain physics insight





Assumptions

- $a \ll L$ (slender screw pinch)
- $|\partial \xi / \partial z| \ll 1$

Kink instability (cont.)

- Use linearised ideal MHD (**see L5 & E10-2**) neglecting pressure terms and assuming constant ρ → equation of motion

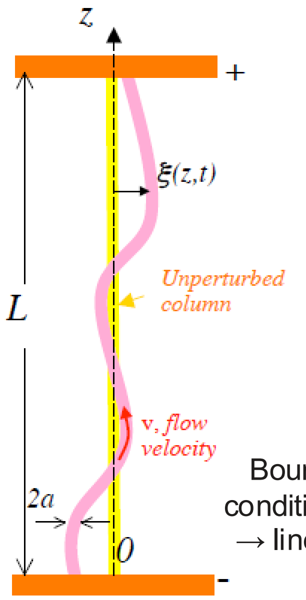
$$\rho \frac{\partial^2 \xi_{x,y}}{\partial t^2} = \frac{B_z^2}{\mu_0} \frac{\partial^2 \xi_{x,y}}{\partial z^2} \mp \frac{B_z B_\varphi}{\mu_0 a} \frac{\partial \xi_{y,x}}{\partial z}$$

with $\eta = \xi_x + i\xi_y$ and $k_0 = B_\varphi/(aB_z)$

$$\frac{\partial^2 \eta}{\partial t^2} = v_A^2 \left(\frac{\partial^2 \eta}{\partial z^2} + i k_0 \frac{\partial \eta}{\partial z} \right)$$

with $v_A = B_z/\sqrt{\mu_0 \rho}$

- Normal mode approach $\eta \propto e^{-i\omega t}$ yields $-\omega^2 \eta = v_A^2(\eta'' + i k_0 \eta)$



Boundary condition η=0 → line tying

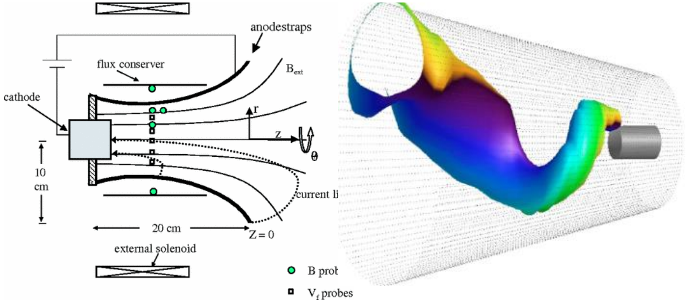
- Unstable solutions (ω^2<0) for $I > I_{K-S} = \frac{4\pi^2 a^2 B_z}{\mu_0 L}$ with growth rate $\mathrm{Im}(\omega) = \frac{\pi v_A}{L}\sqrt{\frac{I^2}{I_{K-S}^2} - 1}$

Kruskal-Shafranov conditions

Universality of the kink instability

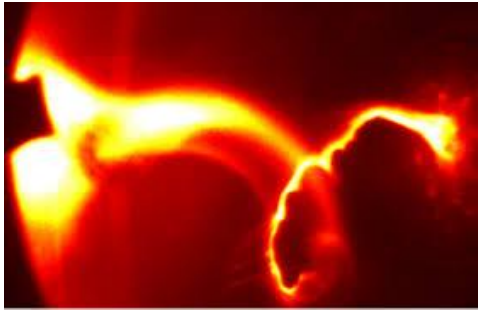
Plasma thrusters

M. Zuin, PRL 2004;
F. Bonomo, PoP 2005

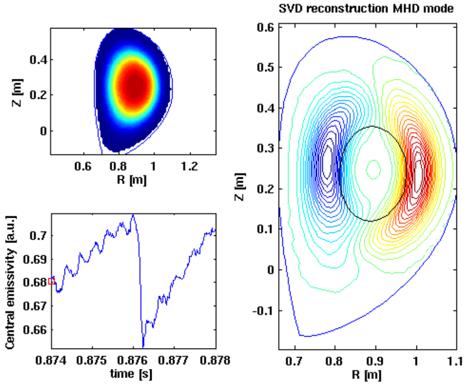


Spheromak formation

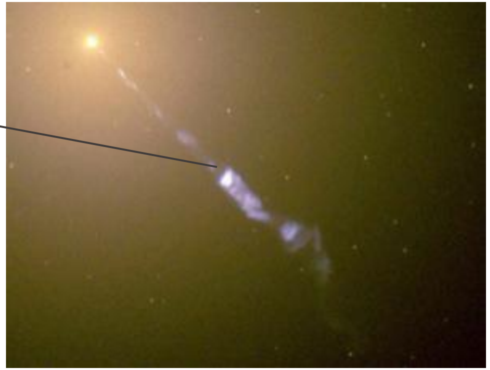
S.C. Hsu, PRL 2004



Sawteeth in tokamaks

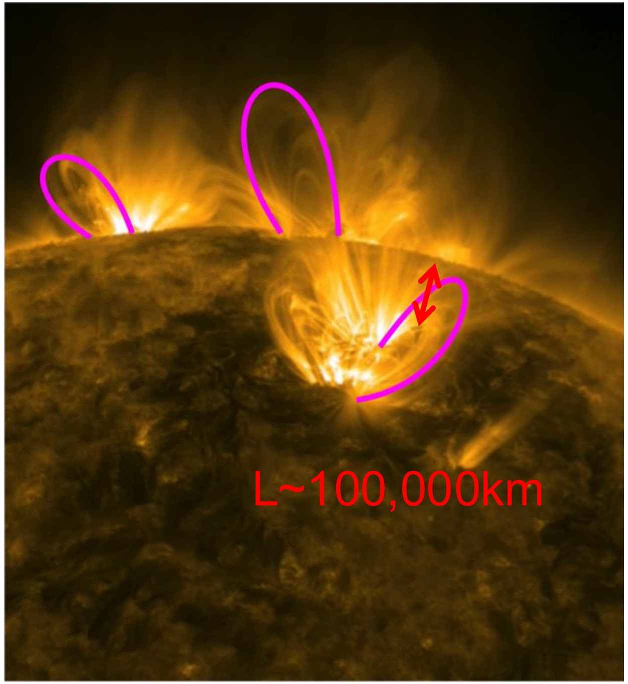


kinked flux rope



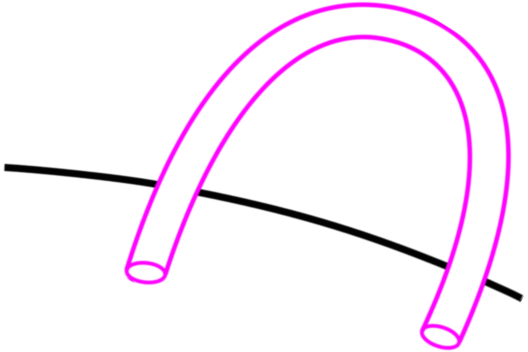
A jet of charged particles shoots out of the galaxy M87

Where is the energy coming from?



Thermal energy ~10^{22} Joules

Coronal mass ejection
~10^{24} Joules
~10^{3} seconds



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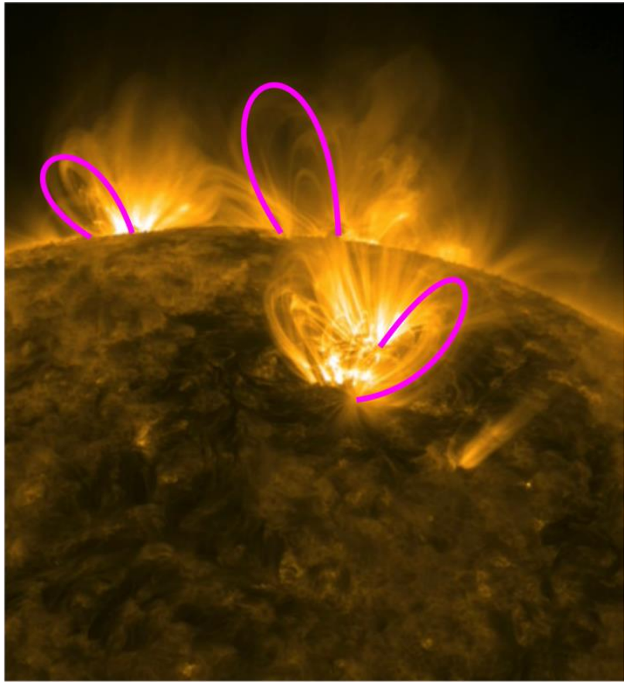
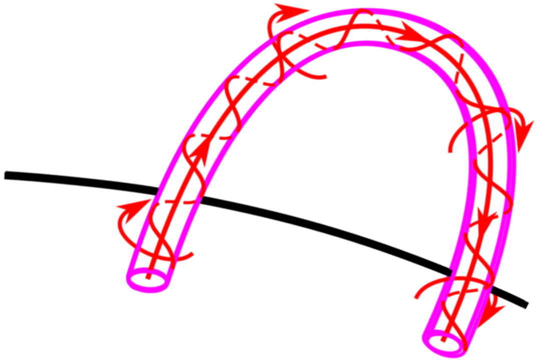


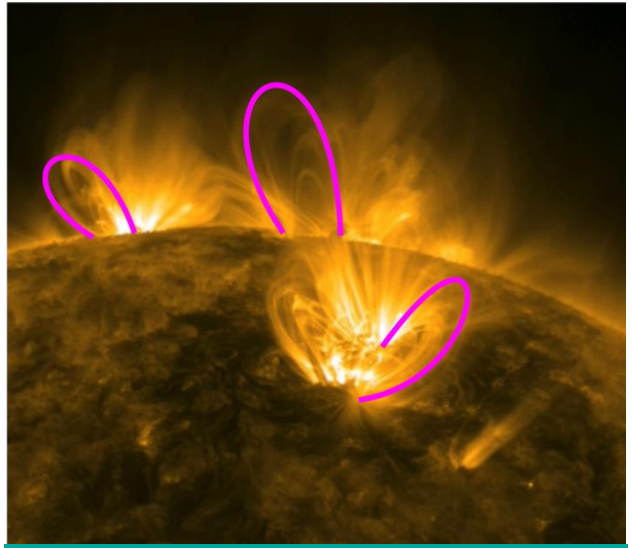
Thermal energy ~10^{22} Joules

Coronal mass ejection
~10^{24} Joules
~10^{3} seconds



Magnetic field

Where is the energy coming from?

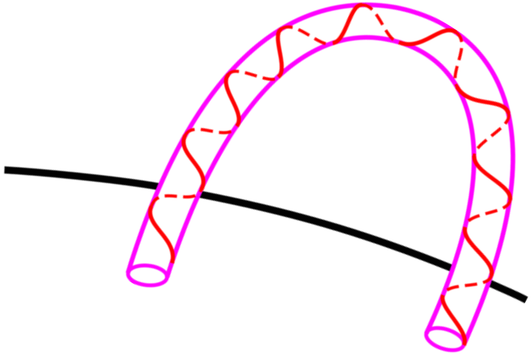


How can magnetic energy be released?

$$\tau_\sigma = \mu_0 \sigma L^2 \sim 10^{14}\mathrm{s} \gg 10^3\mathrm{s}$$

Thermal energy ~10^{22} Joules

Coronal mass ejection
~10^{24} Joules
~10^3 seconds



$$\beta = \frac{\text{Thermal energy}}{\text{Magnetic energy}} \sim 1\%$$

Magnetic reconnection: the early history

1908: discovery of magnetic fields in sunspots (**G.E. Hale**)

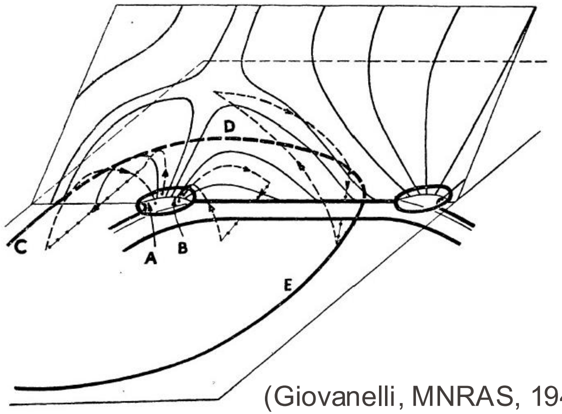
1910's–1940's: MHD not yet developed; Sun described by hydrodynamic

1942–1943: birth of MHD (**H. Alfvén**): frozen-field theorem, Alfvén waves

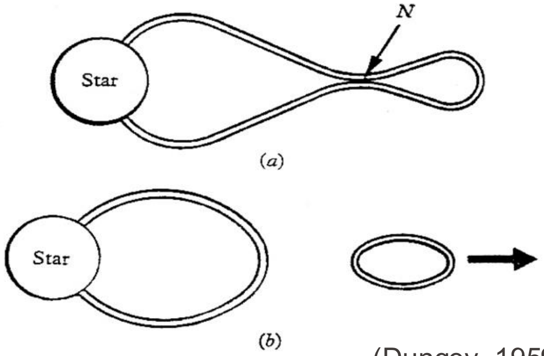
1947: first electromagnetic theory of flares by **R. Giovanelli**: sunspot's field cancels at a **neutral point,** where electric fields can accelerate particles and drive currents

1950's: non-zero resistivity allows the topology of magnetic field to change near the neutral point

The term **magnetic reconnection** is coined by **J. Dungey**: the neutral point is site of a "discharge" whose effect "is to 'reconnect' the line forces"



(Giovanelli, MNRAS, 1947)



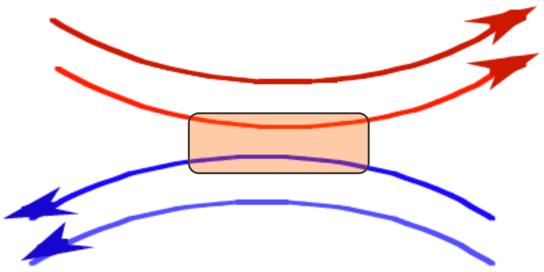
(Dungey, 1959)

Magnetic reconnection: a definition

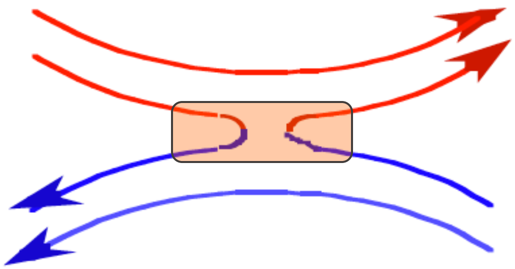
- In most of the universe, the magnetic Reynolds number R_m is large and the magnetic field is attached to the plasma (ideal MHD)

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

- In very thin regions, where magnetic field gradients are very large, magnetic field can slip through the plasma and reconnect



Before reconnection

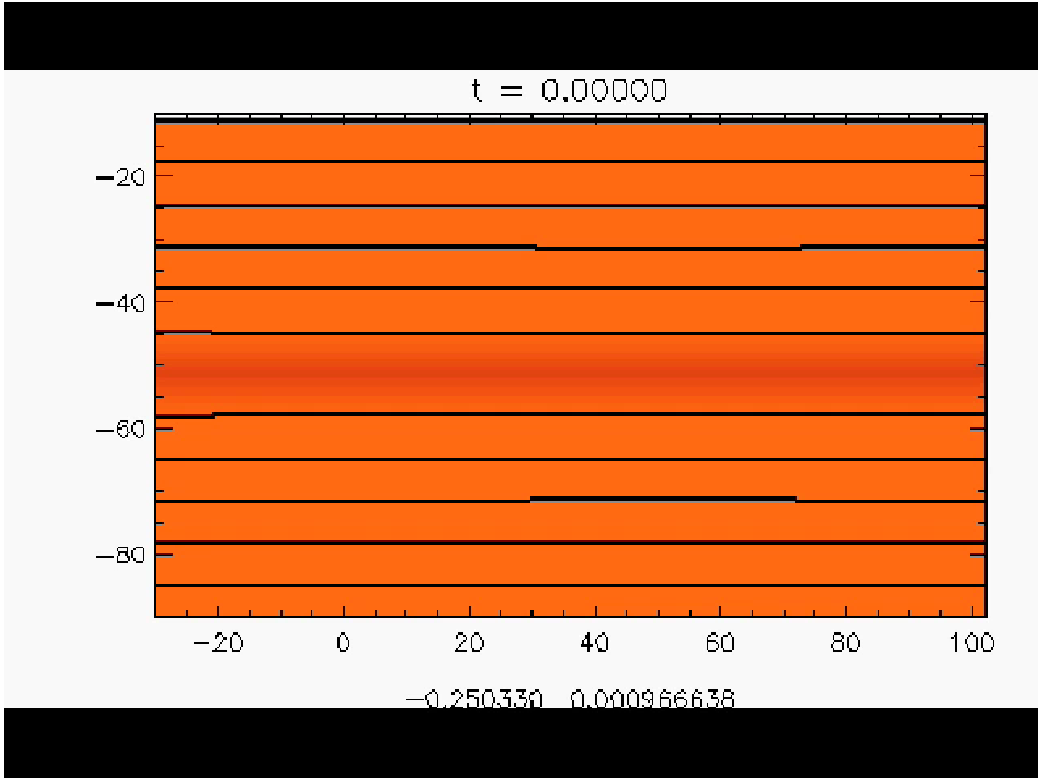


After reconnection

➢ The global magnetic field topology changes, affecting the path of particles and heat

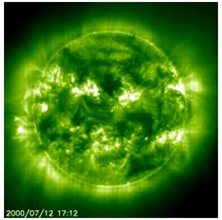
➢ Magnetic energy is converted into heat and kinetic energy

Magnetic reconnection: MHD simulation

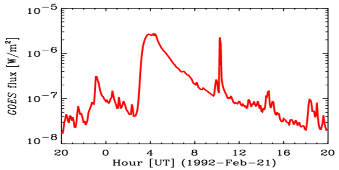


Reconnection occurs very fast after build-up phase

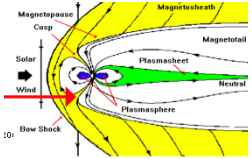
Solar flare



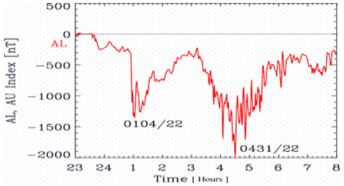
X-ray intensity



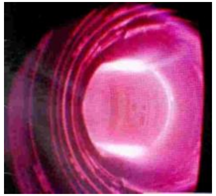
Magnetospheric Aurora-substorm



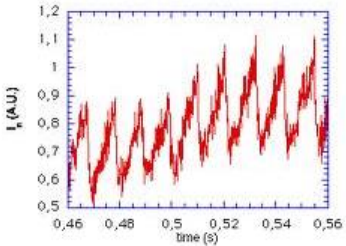
Magnetic Field strength



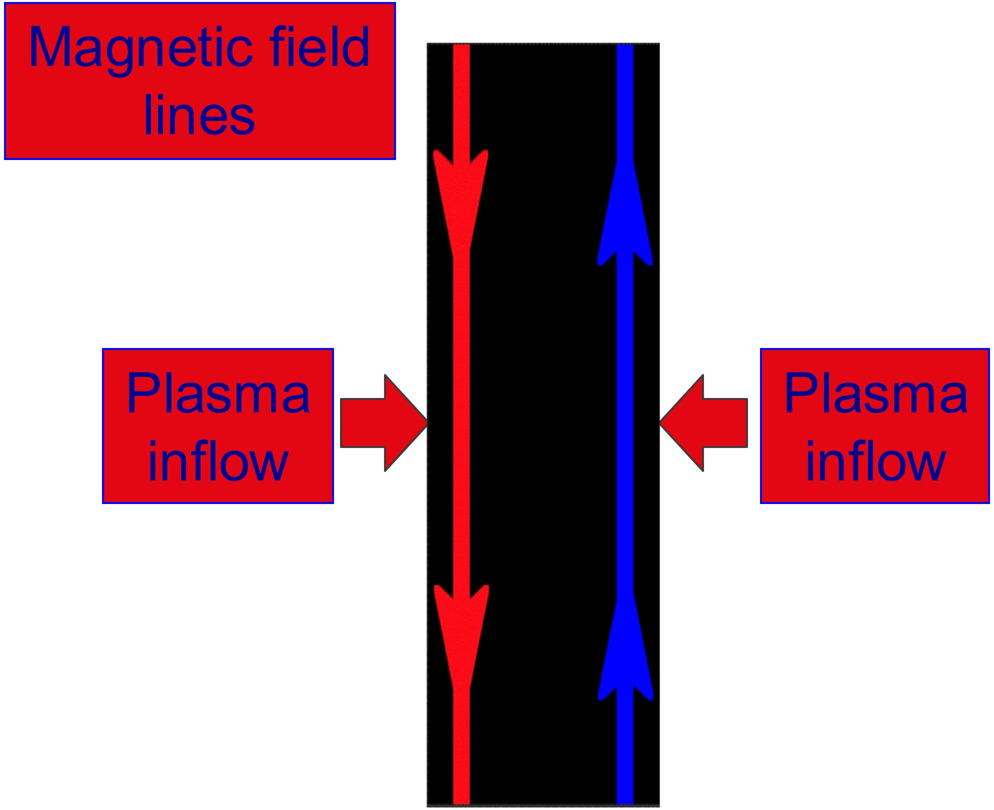
Tokamak Sawtooth



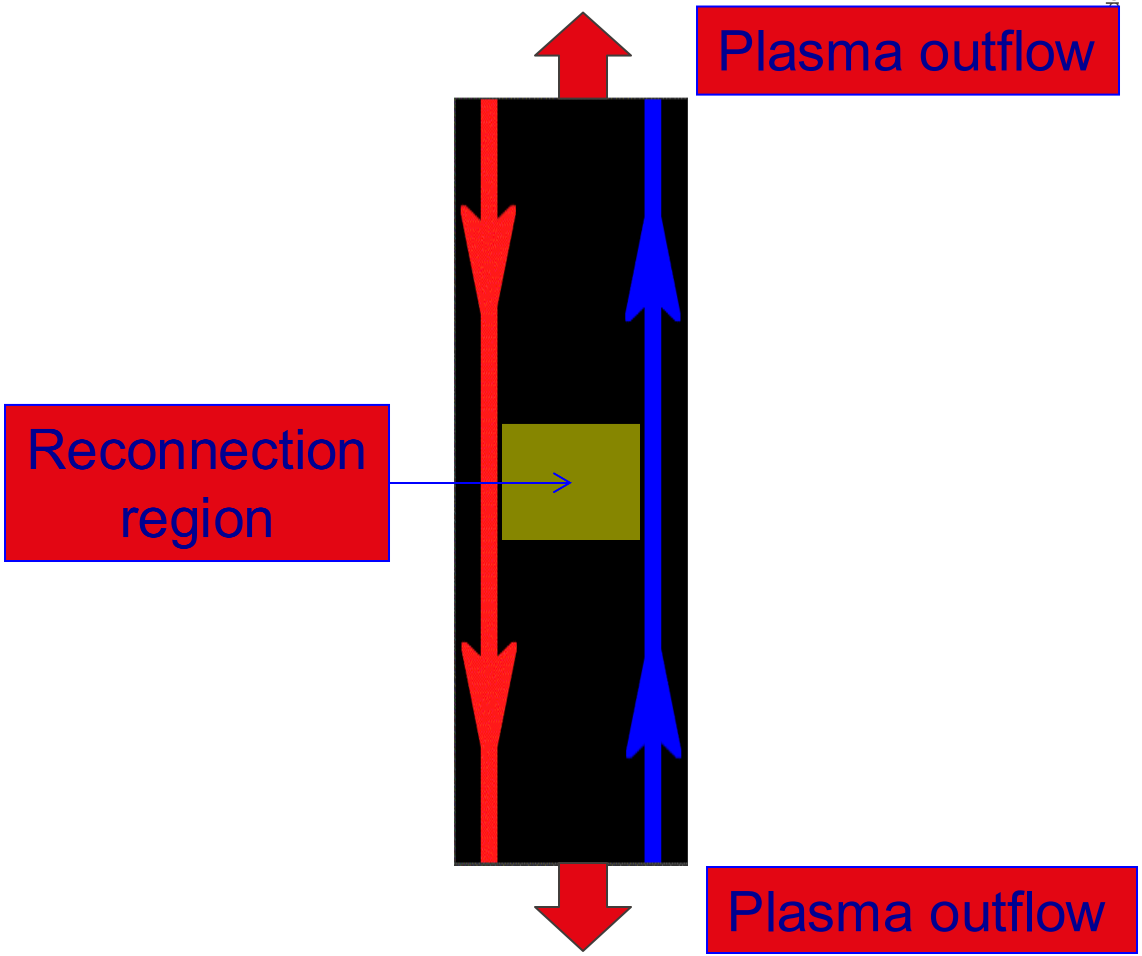
Soft X-ray



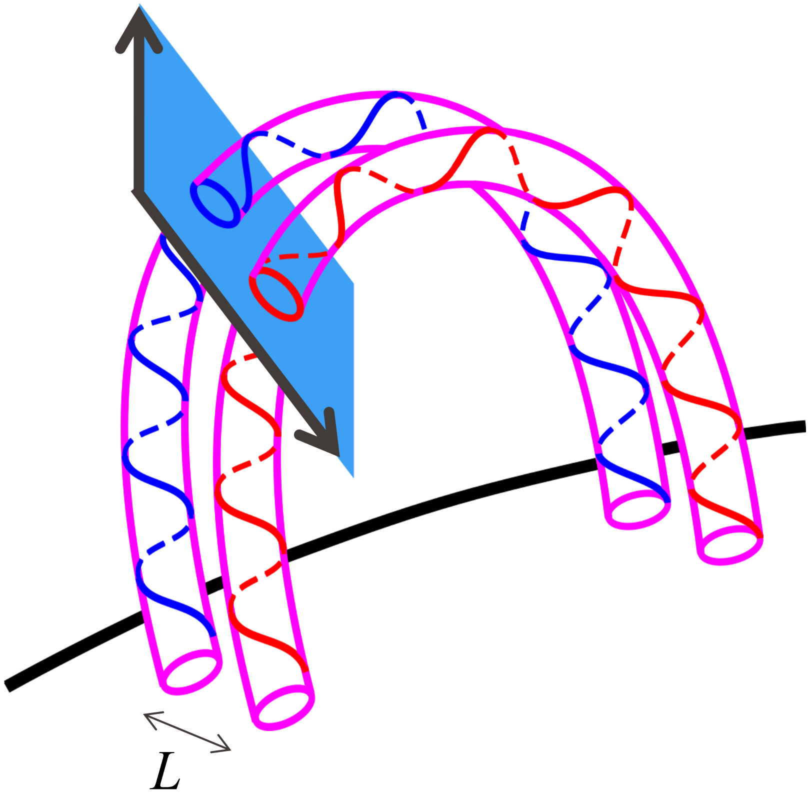
Magnetic reconnection

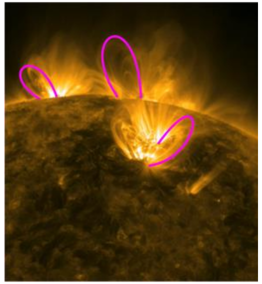


Magnetic reconnection



How can reconnection happen between flux ropes?





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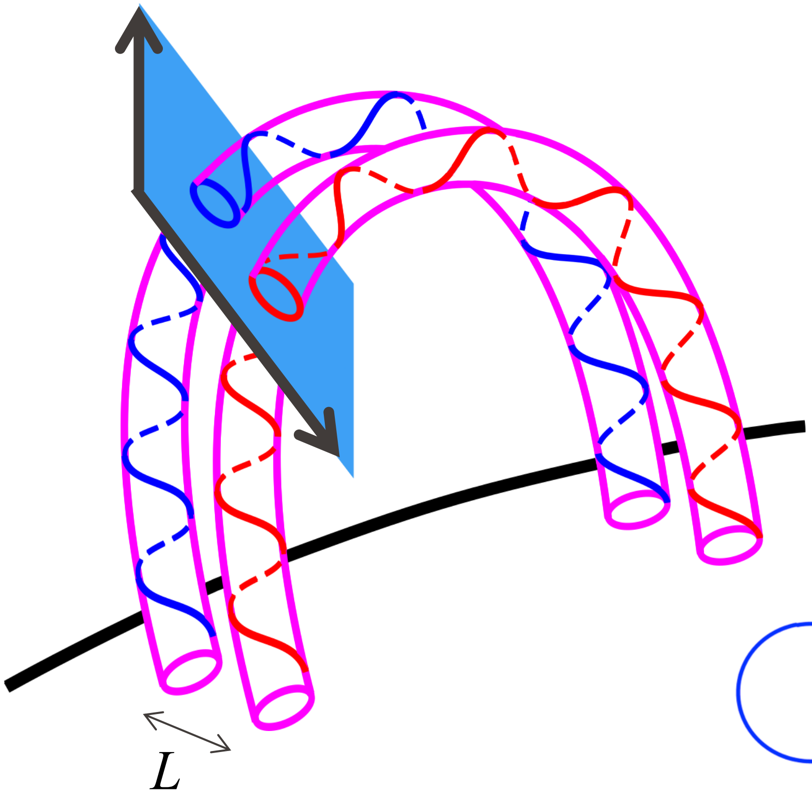

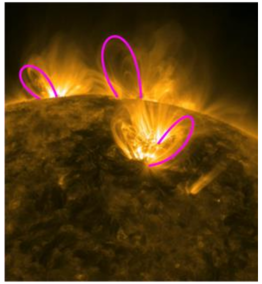

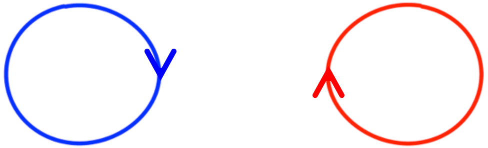

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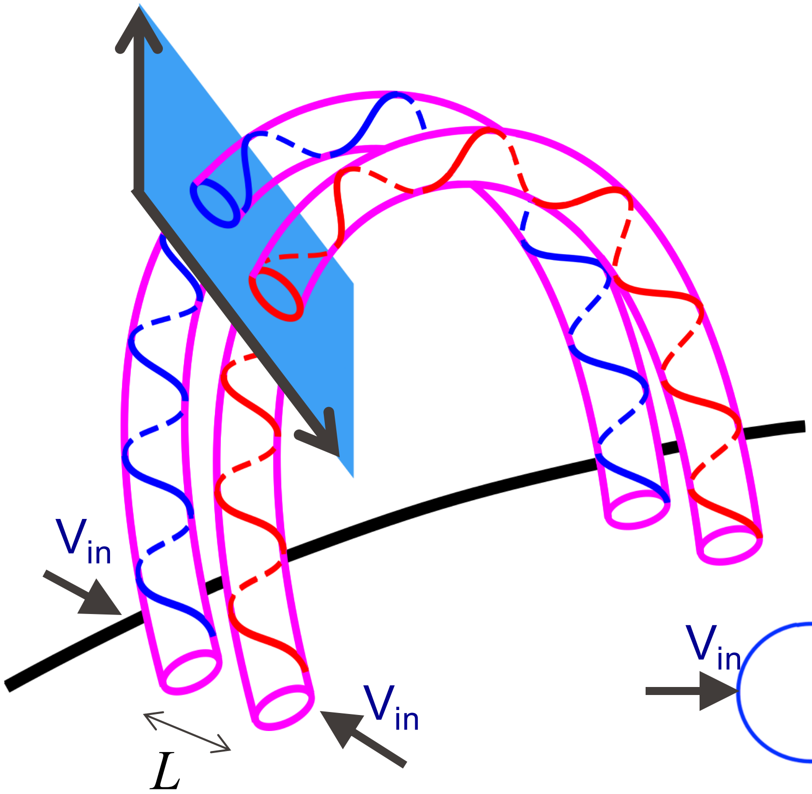

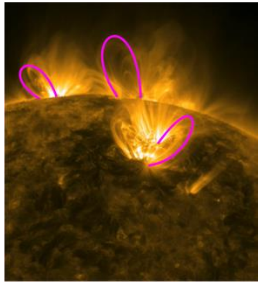

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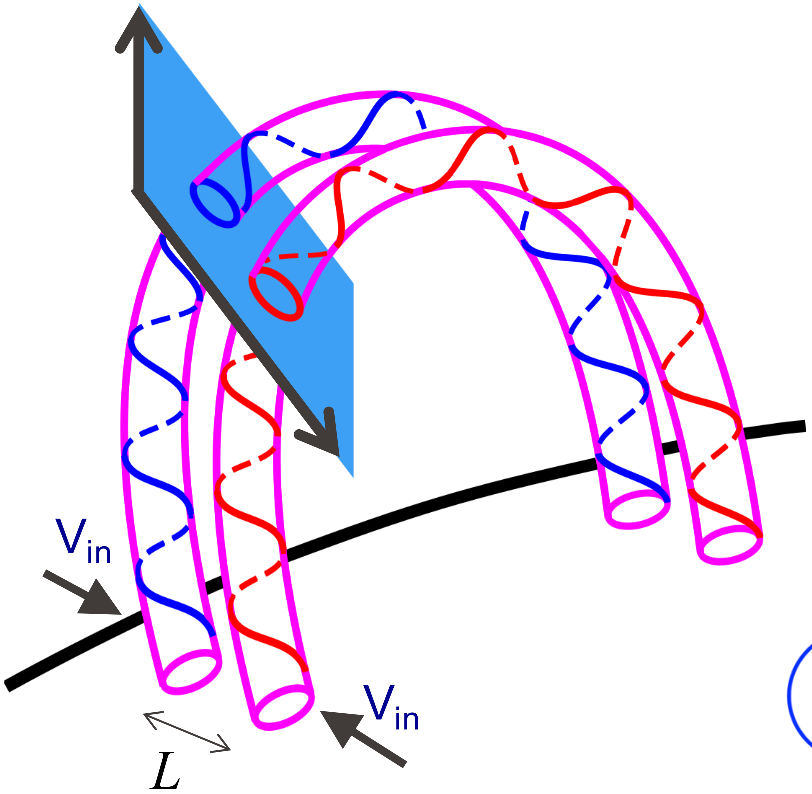

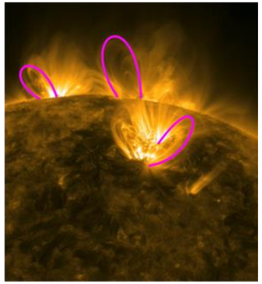

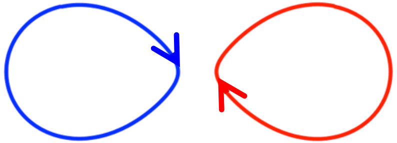

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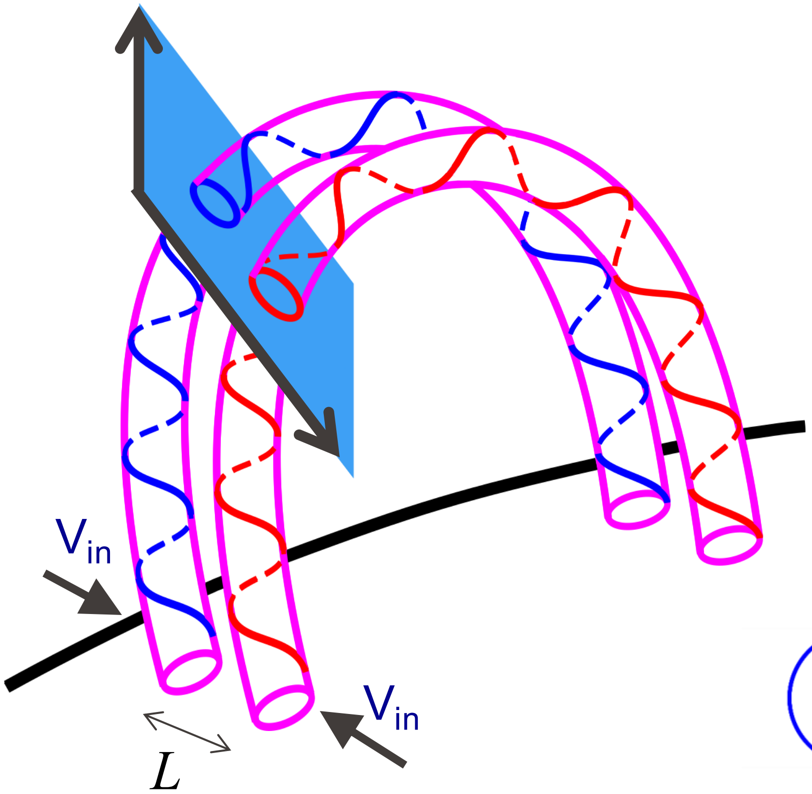
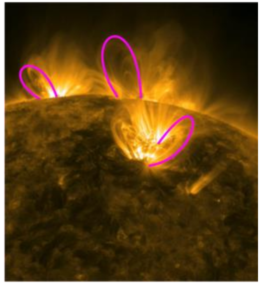
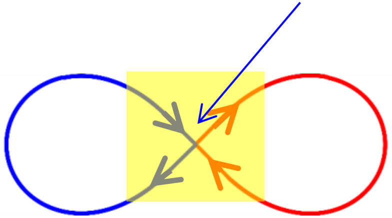




Reconnection region



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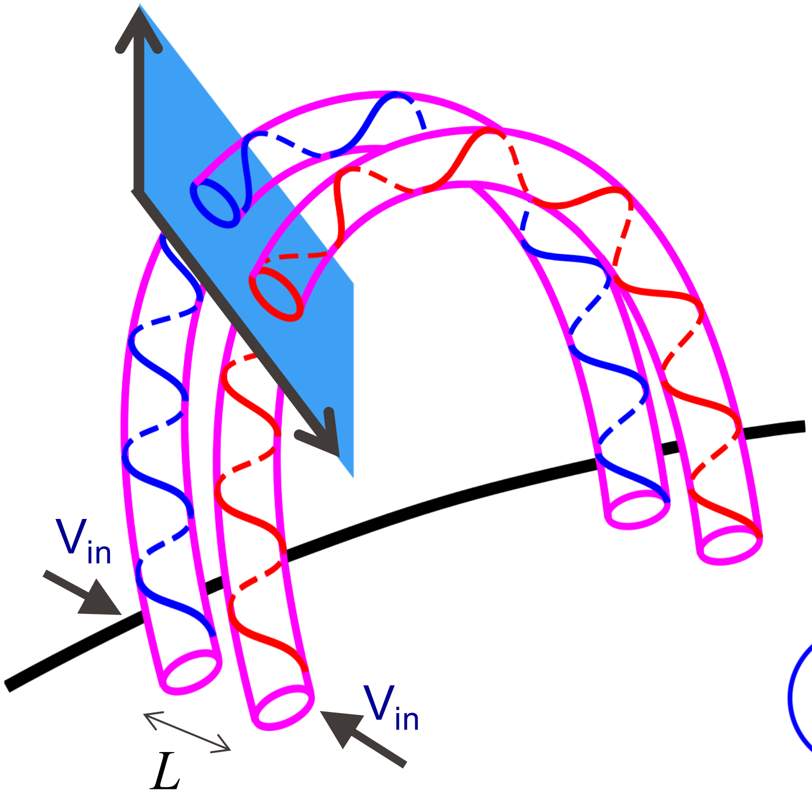
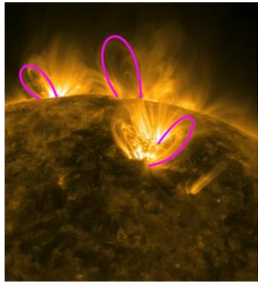
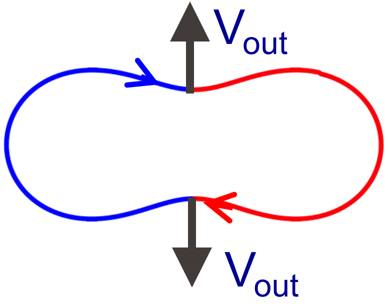
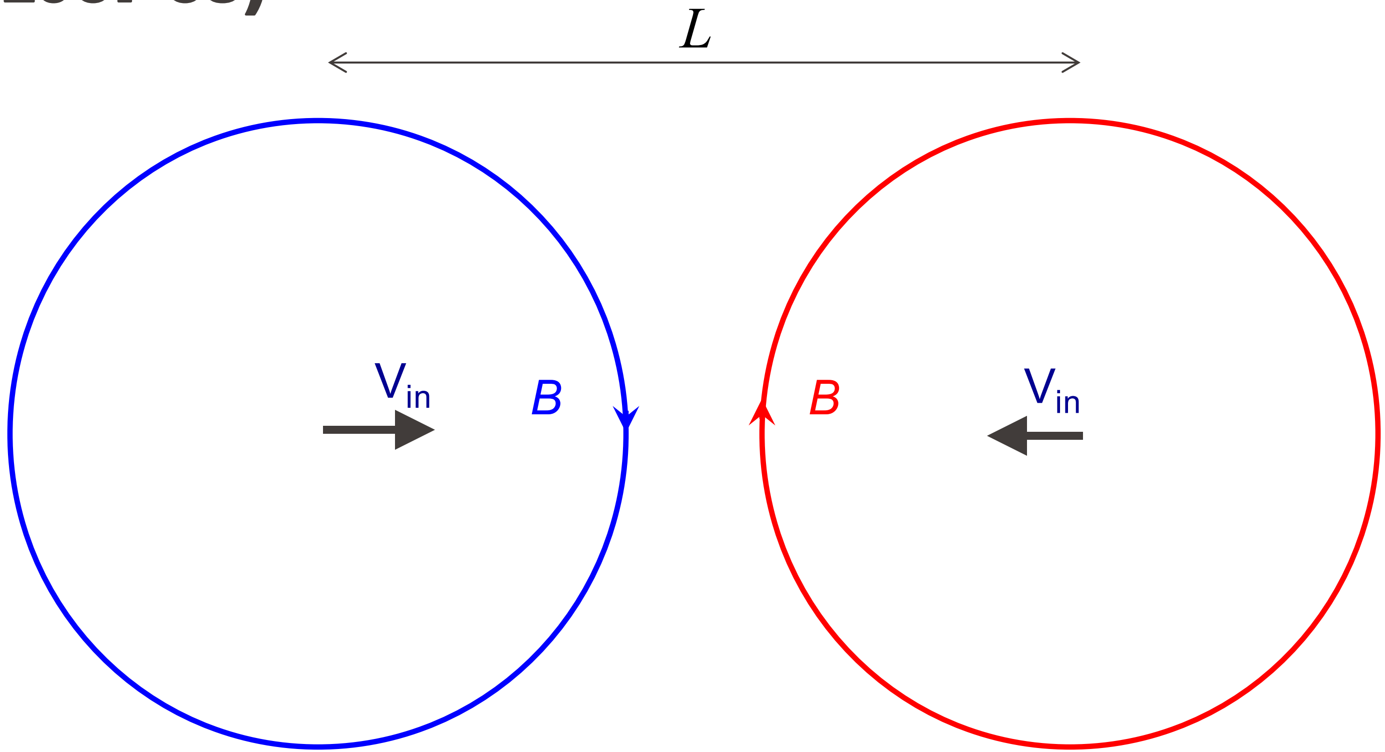




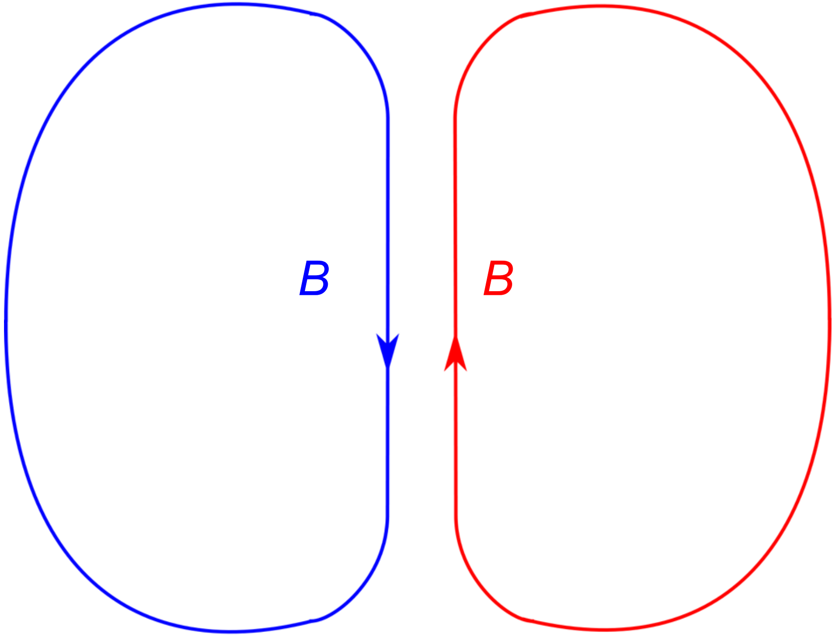


A simple model: Sweet-Parker (1957-58)

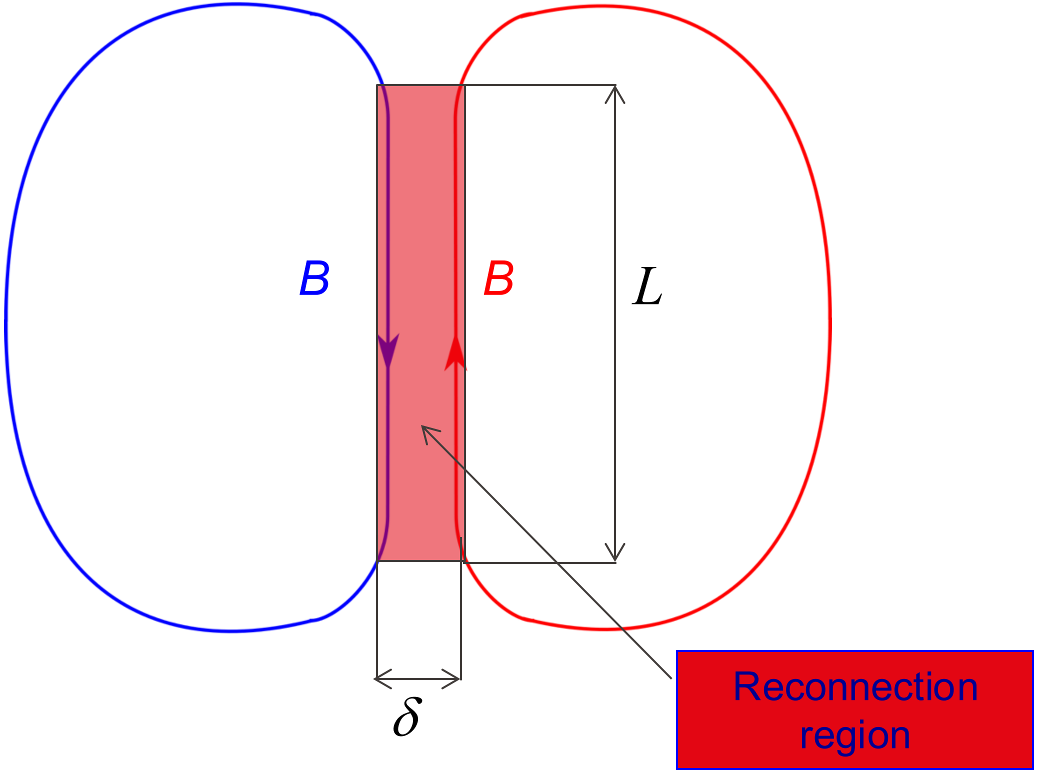


Time scale: $\tau_{\mathrm{SP}} \sim \frac{L}{V_{\mathrm{in}}}$

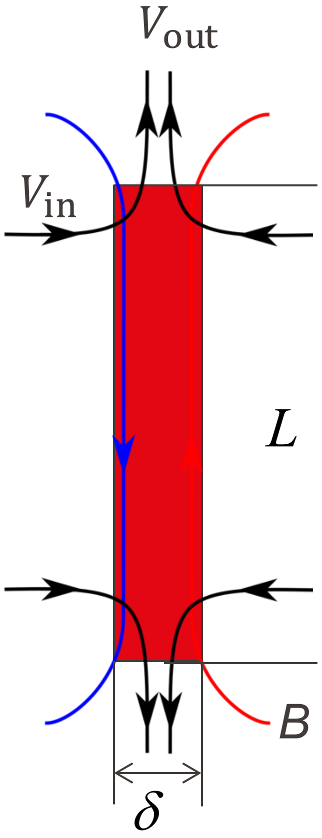
A simple model: Sweet-Parker (1957-58)



A simple model: Sweet-Parker (1957-58)

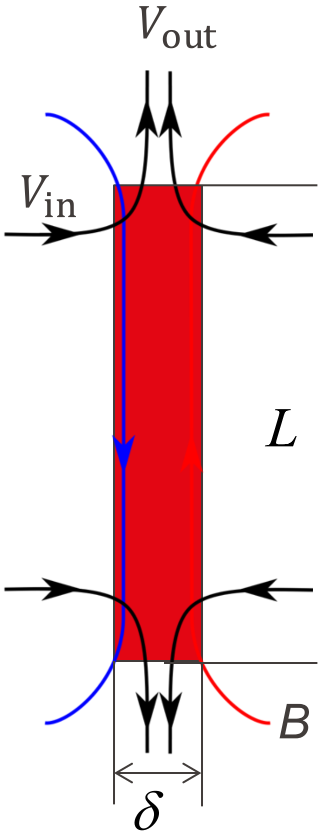


Sweet-Parker model



- Assumptions
  - Steady-state: $\partial_t = 0$
  - Slab geometry: $\partial_y = 0$
  - Incompressible: $\rho = \text{const.}$
  - Low beta: $p \approx 0$
  - Large R_m outside the reconnection sheet $\eta = 0$

Sweet-Parker model (cont.)



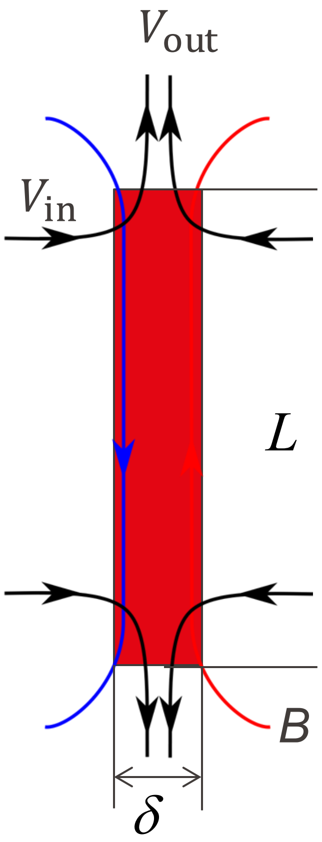
- Continuity equation

$$\nabla \cdot \rho \mathbf{V} = \mathbf{0}$$

- Ohm's law

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{j}$$

Sweet-Parker model (cont.)



- Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

Sweet-Parker model (cont.)

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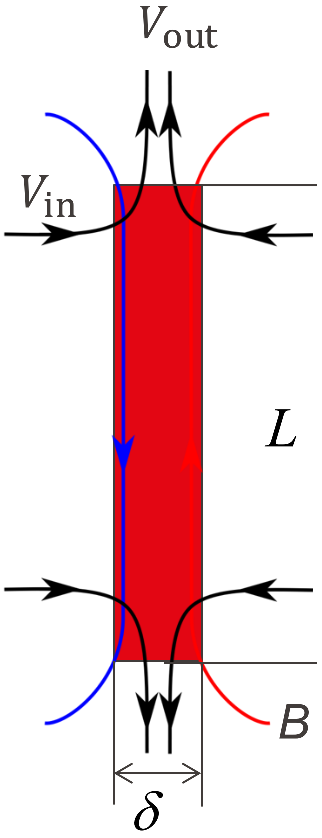
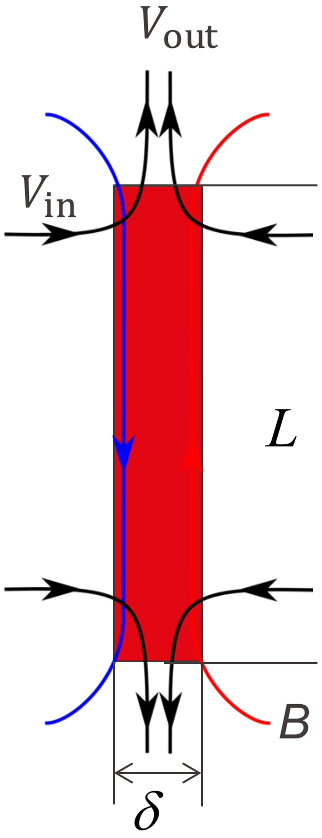


- Equation of motion

$$\rho(\mathbf{V}\cdot\nabla)\mathbf{V} = -\boldsymbol{\nabla}\boldsymbol{p} + \mathbf{j}\times\mathbf{B}$$

Sweet-Parker model (cont.)



- Lundquist number

$$S \equiv \mu_0 \sigma v_A L$$

Energy balance in the SP-model

- The sheet is thin and elongated: $d/L = 1/\sqrt{S}$
- The plasma exits the sheet with Alfven speed: $V_{\text{out}} = V_{\text{A}}$
- The magnetic field strength at the sheet exit is $B_{\text{out}} = \frac{B_0}{\sqrt{S}}$

Further considerations

- Inflow of electromagnetic energy given by the Poynting flux $\bar{S} = \frac{\bar{E}\times\bar{B}}{\mu_0}$
  - With $E = V_{\text{in}}B_0$

$$\frac{EB_0}{\mu_0} L\Delta y = V_{\text{in}} \frac{B_0^2}{\mu_0} L\Delta y \rightarrow \frac{\text{Inflowing kinetic energy}}{\text{Inflowing magnetic energy}} = \frac{\frac{1}{2}\rho V_{\text{in}}^2 \; V_{\text{in}}L}{\frac{B_0^2}{\mu_0} \; V_{\text{in}}L} = \frac{V_{\text{in}}^2}{2V_{\text{A}}^2}$$

➢ Most of the inflowing energy is magnetic!

Energy balance in the SP-model (cont.)

- Estimate outflow of magnetic energy:

$$\frac{EB_{\text{out}}}{\mu_0}d \quad \ll \quad \text{inflowing magnetic energy} \quad \frac{EB_{\text{in}}}{\mu_0}L$$

since $B_{\text{out}} \ll B_{\text{in}}$ and $d \ll L$

$$\longrightarrow \quad \frac{\text{Outflowing kinetic energy}}{\text{Inflowing magnetic energy}} = \frac{\frac{1}{2}\rho V_{\text{out}}^2\, V_{\text{out}} d}{\frac{B_0^2}{\mu_0}\, V_{\text{in}} L} = \frac{\frac{1}{2}V_{\text{out}}^2}{V_A^2} = \frac{1}{2}$$

➢ Half of the incoming magnetic energy is converted into kinetic energy!

Where does the other half go?

Energy balance in the SP-model (cont.)

- Estimate outflow of magnetic energy:

$$\frac{EB_{\text{out}}}{\mu_0}d \quad \ll \quad \text{inflowing magnetic energy} \quad \frac{EB_{\text{in}}}{\mu_0}L$$

since $B_{\text{out}} \ll B_{\text{in}}$ and $d \ll L$

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➢ Half of the incoming magnetic energy is converted into kinetic energy!

Where does the other half go?

- Converted into thermal energy (Joules dissipation) → the reconnection layer is a source of hot and fast plasma with an approximate equipartition between flow and thermal energy

How fast is Sweet-Parker reconnection?

- Typical Sweet-Parker reconnection times

$$\tau_{\mathrm{rec-SP}} \sim \frac{L}{V_{\mathrm{in}}} \approx \frac{L}{V_{\mathrm{A}}} \sqrt{S}$$

	L	B	n	v_A	T	$\tau_{rec\text{-}SP}$
Solar flare	10^5km	0.1-1T	2.5×10^{15}m^{-3}	10^6m/s	10^7K	10^7s
Tokamak	1m	0.1T	10^{19}m^{-3}	10^6m/s	1KeV	1-10ms

- Typical flares happen on time scales of 100s → Sweet-Parker reconnection is still too slow
- Typical sawteeth happen on time scales of 10-50µs → Sweet-Parker reconnection is still too slow

Brief summary: Sweet & Parker model

- First reconnection model: a diffusion region with $d \ll L$, steady state, 2D, Spitzer resistivity

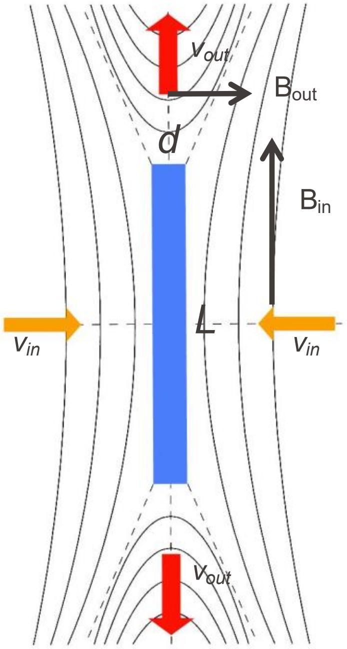
➢ Resulting reconnection times still too slow to explain observations (sun & tokamak alike)

$$\tau_{\mathrm{rec-SP}} = \frac{L}{V_A}\sqrt{\frac{\mu_0 V_A L}{\eta}}$$

Modify the sheet geometry

Increase the resistivity

Introduce new physics: beyond single-fluid MHD, 3D physics



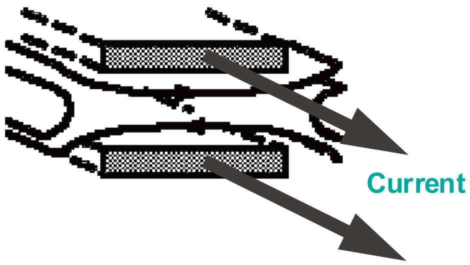
Open questions in magnetic reconnection

- What causes fast magnetic reconnection?
- What is the interplay between small-scale physics and global dynamics?
- What can astrophysicists learn about reconnection from laboratory experiments and near-Earth space plasmas?

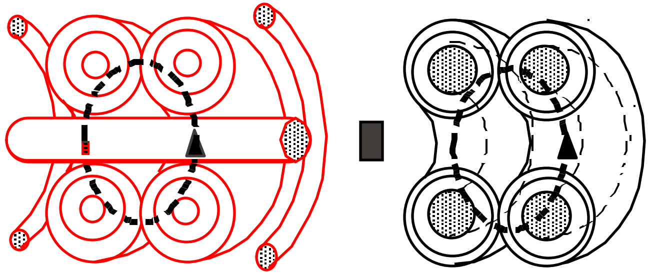
Laboratory experiments to investigate magnetic reconnection

H. Reimerdes

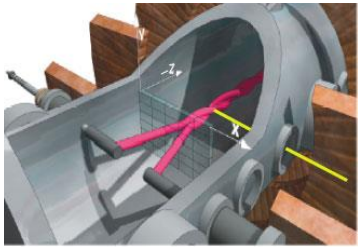
Stenzel and Gekelman – USA, Frank- Russia, Grulke and Klinger - Europe



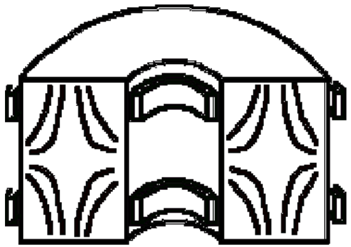
Yamada and Ji – USA, Brown – USA, Ono - Japan



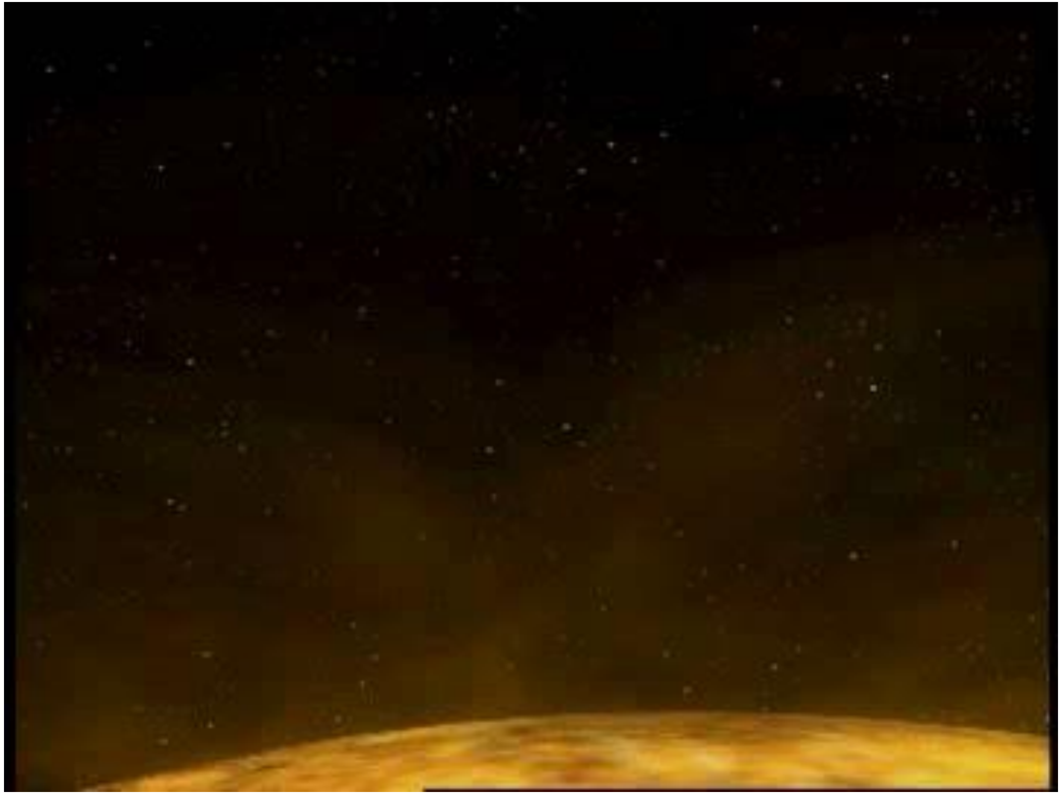
Furno and Intrator – USA, Gekelman - USA



Fasoli and Egedal – USA



Magnetic storms



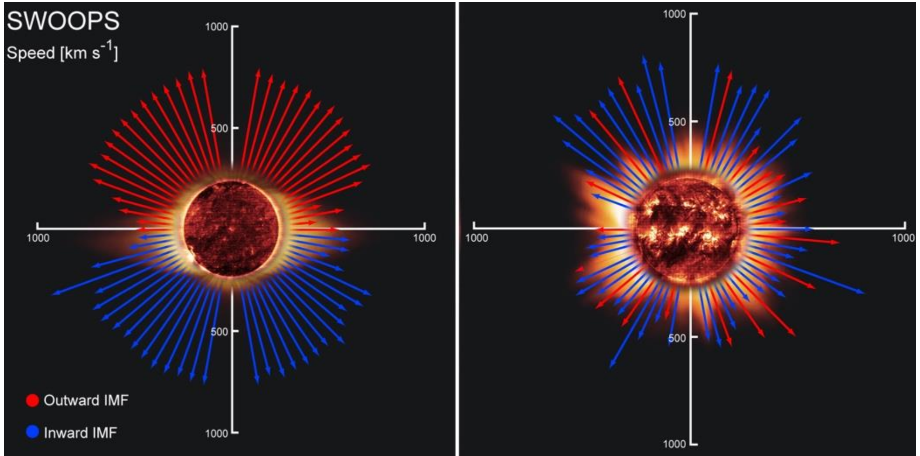
First hints of solar wind

- Birkeland (1908) suggested continuous particle emission from sunspots to explain correlation between sunspots and auroras
- Chapman and Ferraro (1931) suggested that particles are emitted during flares with space otherwise being empty
- Bierman (1951) recognised that cometary tails point directly away from the Sun regardless of the comet's velocity and direction → ionised gas pushed away by the solar wind

Spacecrafts directly measure the solar wind

- Mariner 2 (launched 1962) – first successful mission for planetary exploration yielded 3 months of solar wind data while traveling to Venus → solar wind speed ~300-700km/s
- Ulysses (launched 1990) - performed three sets of polar passes
  - Swoops (Solar Wind Observations Over the Poles of the Sun)

Solar minimum

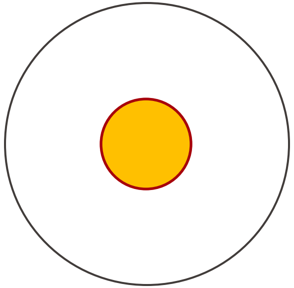


Solar maximum

Source: ESA

Fluid model of the solar wind

Assumptions: spherical symmetry, stationary, Lorentz force negligible, isothermal



- Continuity equation

$$\nabla \cdot \rho \mathbf{V} = \mathbf{0}$$

- Equation of motion

$$\rho(\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p + \mathbf{j} \times \mathbf{B} - G\frac{\rho M_r}{r^2}$$

- Ideal gas law

$$p = \rho \frac{k_B T}{\mu}$$

Fluid model of the solar wind

- Differential equation for the velocity

$$\frac{1}{V}\frac{\partial V}{\partial r}\left(\frac{V^2}{V_s^2}-1\right)=\frac{2}{r}\left(1-\frac{GM_\odot}{2V_s^2 r}\right)$$

Some typical values of the solar wind

- Critical radius $$r_c = \frac{GM_\odot}{2V_s^2}$$

- (Isothermal-) Sound speed $$V_s = \sqrt{\frac{k_B T}{\mu}}$$

The solar wind velocity

- Integrate equation of motion along flow (assuming isothermal plasma)

$$\frac{V^2}{2} + V_s^2 \log \rho + \phi_G = const.$$

  - Use conservation of mass $\rho V r^2 = \rho_0 V_0 r_0^2$

➢ Velocity at radius *r*

$$\frac{V^2 - V_0^2}{2} = -V_s^2 \log\left(\frac{V_0 r_0^2}{V r^2}\right) + \phi_{G0} - \phi_G$$

$$\Rightarrow \qquad \frac{V}{V_0} e^{-\frac{V^2}{2V_s^2}} = \frac{r_0^2}{r^2} e^{\left[a\left(1-\frac{r_0}{r}\right)-\frac{V_0^2}{2V_s^2}\right]}$$

The solar wind velocity (cont.)

- Super-sonic wind: For $V(r_c) = V_s$ (using $r_c = a\frac{r_0}{2}$)

$$\frac{V_0}{V_s} e^{-\frac{V_0^2}{2V_s^2}} = \frac{a^2}{4} e^{-a+\frac{3}{2}}$$

- With $r_c \approx 4.5 R_\odot$ → $a \approx 9$

$$\frac{V_0}{V_s} \ll 1 \qquad \Rightarrow \qquad \frac{V_0}{V_s} \approx \frac{a^2}{4} e^{-a+\frac{3}{2}}$$

- One can equally estimate

$$V \approx 2V_s \log\left(\frac{r}{r_c}\right)^{\frac{1}{2}} \quad \text{for} \quad r \to \infty$$

Some numbers

- Initial velocity of super-sonic wind

$$V_0 \approx V_s \frac{a^2}{4} e^{-a+\frac{3}{2}} \qquad V_0 \approx 0.011\, V_s$$

- Mass-loss rate

$$\frac{dM}{dt} \approx 4\pi\rho_0 V_0 R_\odot^2 \qquad \approx 1.5 \times 10^9\ \mathrm{kg\,s^{-1}}$$

- Solar wind at earth ($\mathrm{r} = 1\mathrm{AU} \approx 214 R_\odot \approx 48 r_c$)

$$V \approx 2V_s \log\left(\frac{r}{r_c}\right)^{\frac{1}{2}} \qquad \approx 4V_s \qquad \approx 5.6 \times 10^5\ \mathrm{m/s}$$